On the identifiability of household production functions with joint products

Kooreman, P.; Kapteyn, A.J.

Publication date:
1987

UNIVERSITEIT BRABANT
POSTBOX 90153
5000 LE TILBURG
THE NETHERLANDS

[redacted]

DEPARTMENT OF ECONOMICS

RESEARCH MEMORANDUM

[redacted]

ON THE IDENTIFIABILITY OF HOUSEHOLD PRODUCTION FUNCTIONS WITH JOINT PRODUCTS: A COMMENT

Peter Kooreman, Arie Kapteyn

FEW 254

May 1987

Department of Econometrics
Tilburg University
P.O. Box 90153
5000 LE Tilburg
The Netherlands

1. Introduction

Since the appearance of the classical paper by Becker (1965) on household production theory, several attempts have been made to estimate household production functions. To disentangle empirically the effects of the household's preferences and its production technology, either the household products have to be observable, which is rarely the case,[1] or one has to assume that the household products can also be bought on the market. In the latter case preferences hardly play a role, because the perfect substitutability of the household products output for market goods generates efficiency conditions that relate the number of hours spent on homework by each spouse to their market wages, at least if they work in a paid job. Having data on time spent on homework, wages and non-labor income, (some) production function parameters can then be estimated. Examples are the papers by Gronau (1977, 1980) and a recent paper in this Review by Graham and Green (1984) (G & G in the sequel).

A particular feature of the G & G model is that it allows for joint production, meaning that the spouses may enjoy homework and hence count part of the time spent on it as pure leisure. After choosing specific functional forms G & G derive an equation for female homework which they estimate by OLS using cross-section data on 921 two-earner households. Since their model is underidentified different sets of additional restrictions on structural parameters are imposed. These different sets of restrictions yield dramatically different estimates of the value of home production.

The ostensible underidentification in G & G's model emanates from ignoring the information contained in the corresponding male homework equation. In Section 2 we show that if the equations for female homework and for male homework are considered jointly, the model is in fact overidentified.

However, the possibility to identify the parameters of the household production function and the parameters describing the extent of joint production does not only require the assumption that the output could also

---

1) An exception is the paper by Rosenzweig and Schultz (1983) who use birth weight as an indicator of the output of the household health production function.

be bought on the market. It also requires the use of specific, restrictive functional forms as the ones employed by G & G. In Section 3 we show that in general a model with joint production is fundamentally distinguishable from a model without joint production. So, in G & G's model conclusions on joint production will entirely rest upon assumptions about functional form.

2. Identification in the G & G model

The model formulated by Graham and Green consists of the following six relations

$$U = U(C,\ M_hL_h,\ M_wL_w) \tag{1}$$

$$C = X_m + Z \tag{2}$$

$$Z = Z(X_z,\ M_h^aH_h,\ M_w^bH_w) \tag{3}$$

$$X_m + X_z = W_hN_h + W_wN_w + v \tag{4}$$

$$\ell_i + H_i + N_i = T;\ i = h,w \tag{5}$$

$$L_i = \ell_i + g_i(H_i);\ i = h,w \tag{6}$$

Here U(.) is the household utility function. C represents goods which are either obtained in the market (X_m) or produced at home (Z). Eq. (3) describes the production of Z within the household, where X_z equals market purchased inputs used in production and $M_h^aH_h$ and $M_w^bH_w$ are the "effective" time inputs of husband and wife, respectively. Eq. (4) is the household budget constraint where v is non-labor income, W_h and W_w are hourly wages, and N_h and N_w are hours of work in paid jobs by the husband and wife, respectively. The function $g_i(H_i)$ is introduced to account for joint production and denotes the fraction of time spent on home production which is valued as pure leisure.

For empirical implementation, G & G choose the following particular functional forms for $Z(X_z,\ M_h^aH_h,\ M_w^bH_w)$ and $g_i(H_i)$:

$$Z = A(M_h^a H_h)^{\gamma_h}(M_w^b H_w)^{\gamma_w} X_z^{\beta} \tag{7}$$

$$g_i(H_i) = H_i - T^{-\delta_i}(1+\delta_i)^{-1} H_i^{1+\delta_i}, \ i = h, w. \tag{8}$$

Next, they show that in order to maximize (1) s.t. (2)-(6), H_h, H_w and X_z should satisfy the following first order conditions:

$$\beta A(M_h^a H_h)^{\gamma_h}(M_w^b H_w)^{\gamma_w} X_z^{\beta-1} = 1 \tag{9}$$

$$\gamma_h A\, M_h^{a\gamma_h} H_h^{\gamma_h - 1}(M_w^b H_w)^{\gamma_w} X_z^{\beta} = W_h \left[\frac{H_h}{T}\right]^{\delta_h} \tag{10}$$

$$\gamma_w A(M_h^a H_h)^{\gamma_h} M_w^{b\gamma_w} H_w^{\gamma_w - 1} X_z^{\beta} = W_w \left[\frac{H_w}{T}\right]^{\delta_w} \tag{11}$$

Note that the utility function is left unspecified and does not play a role in the system (9)-(11); due to the assumption of perfect substitutability between home products and market goods, H_m, H_g and X_z are completely determined by the conditions for efficient production of Z, at least for exogenous W_m and W_w.

Solving the system simultaneously for H_w and H_h and expressing the solution in logs yields

$$\begin{aligned}\log H_w = {} & c_w + q_w^{-1} \log A + q_w^{-1}\left[\frac{\gamma_h}{1+\delta_h} + \beta - 1\right] \log W_w \\ & - q_w^{-1}\left[\frac{\gamma_h}{1+\delta_h}\right] \log W_h + q_w^{-1} a\gamma_h \log M_h + q_w^{-1} b\gamma_w \log M_w\end{aligned} \tag{12}$$

and

$$\begin{aligned}\log H_h = {} & c_h + q_h^{-1} \log A + q_h^{-1}\left[\frac{\gamma_w}{1+\delta_w} + \beta - 1\right] \log W_h \\ & - q_h^{-1}\left[\frac{\gamma_w}{1+\delta_w}\right] \log W_w + q_h^{-1} a\gamma_h \log M_h + q_h^{-1} b\gamma_w \log M_w\end{aligned} \tag{13}$$

with

$$q_w = (1-\beta)(1+\delta_w) - \gamma_w - \frac{\gamma_h(1+\delta_w)}{1+\delta_h} \tag{14}$$

and

$$q_h = (1-\beta)(1+\delta_h) - \gamma_h - \frac{\gamma_w(1+\delta_h)}{1+\delta_w} \tag{15}$$

Now consider the following reduced form regressions

$$\log H_w = C_w + k_w \log A + \ell_w \log W_w + m_w \log W_h + n_w \log M_w + o_w \log M_h \tag{16}$$

$$\log H_h = C_h + k_h \log A + \ell_h \log W_w + m_h \log W_h + n_h \log M_w + o_h \log M_h \tag{17}$$

The estimated coefficients in (16) and (17) can be used to obtain estimates for the structural parameters in (12) and (13).

For some unknown reason, G & G ignore the equations for male homework, although their data set seems to contain the information required for the estimation of (17).

Since the unknown parameters γ_h, γ_w, β, δ_h, δ_w, a and b cannot be retrieved from the estimated coefficients in (16) only, G & G have to impose additional restrictions.

However, if one exploits the information from both (16) and (17), the structural parameters can be identified as follows. β can be estimated by either $(k_w + \ell_w + m_w)k_w^{-1}$ or $(k_h + \ell_h + m_h)k_h^{-1}$. Given an estimate for β, estimates for γ_w, γ_h, δ_w and δ_h can be obtained from

$$\frac{\gamma_h}{1+\delta_h} = -\frac{m_w}{k_w}, \qquad k_w^{-1} = (1-\hat{\beta})(1+\delta_w) - \gamma_w - \frac{\gamma_h(1+\delta_w)}{1+\delta_h}$$

$$\frac{\gamma_w}{1+\gamma_w} = -\frac{m_h}{k_h}, \qquad k_h^{-1} = (1-\hat{\beta})(1+\delta_h) - \gamma_h - \frac{\gamma_w(1+\delta_h)}{1+\delta_w}$$

The solution is given by

$$\hat{\delta}_w = k_w^{-1}(1-\hat{\beta} + m_w k_w^{-1} + m_h k_h^{-1})^{-1} - 1$$

$$\hat{\delta}_h = k_h^{-1}(1-\hat{\beta} + m_w k_w^{-1} + m_h k_h^{-1})^{-1} - 1$$

$$\hat{\gamma}_w = -m_h k_h^{-1}(1+\hat{\delta}_w)$$

$$\hat{\gamma}_h = -m_w k_w^{-1}(1+\hat{\delta}_h)$$

Finally, given the estimates for γ_h and γ_w, a can be estimated by $o_w . k_w^{-1} . \hat{\gamma}_h^{-1}$ or by $o_h . k_h^{-1} . \hat{\gamma}_h^{-1}$, whereas b can be estimated by $n_w . k_w^{-1} . \hat{\gamma}_w^{-1}$ or by $n_h . k_h^{-1} . \hat{\gamma}_w^{-1}$.

3. Non-identifiability of joint production in the general model

Reconsider the general formulation (1)-(6) and let us ignore the productivity variables M_h and M_w, i.e. we take $M_h \equiv M_w \equiv 1$. This does not affect the main argument. The assumptions made by G & G regarding the function g_i are

(i) $0 \leq g_i'(H_i) \leq 1$

(ii) $g_i''(H_i) < 0$

(iii) $\lim_{H_i \to 0} g_i'(H_i) = 1$

(iv) $\lim_{H_i \to T} g_i'(H_i) = 0$, $i = h,w$

It is easy to show that the first order conditions determining X_z, H_h and H_w are (cf. Kooreman and Kapteyn, 1987, Appendix A):

$$\frac{\partial Z}{\partial X_z} = 1 \tag{18}$$

$$\frac{\partial Z}{\partial H_i} = W_i[1-g_i'(H_i)], \quad i = h,w \tag{19}$$

Now define

$$H_i^* = H_i - g_i(H_i) \equiv k_i(H_i), \tag{20}$$

as the "pure household production time" (i.e. H_i minus the part that is considered to be leisure by the individual). By construction the function k(.) is monotonically increasing. Let F be the production function that is obtained by substituting $H_i = k_i^{-1}(H_i^*)$ into (3), i.e.

$$Z = F(X_z, H_h^*, H_w^*) \equiv Z(X_z, H_h, H_w) \tag{21}$$

We then have

$$\frac{\partial Z}{\partial H_i} = \frac{\partial F}{\partial H_i^*} \cdot \frac{\partial H_i^*}{\partial H_i} = \frac{\partial F}{\partial H_i^*}[1 - g_i'(H_i)], \quad i = h,w \tag{22}$$

and we can write (5) as

$$L_i + H_i^* + N_i = T, \quad i = h,w \tag{23}$$

The model consisting of the equations (1), (2), (21), (4), (23) is equivalent to the model consisting of (1)-(6). Both models generate identical first order conditions and it is also easy to show that if Z is a quasi-concave twice differentiable function, then so is F. Since (1), (2), (21), (4), (23) is a model without joint production and (1)-(6) a model with joint production, it is clear that in the G & G framework it is fundamentally impossible to identify jointness in household production.

References

Becker, G.S. (1965), "A theory of the allocation of time", The Economic Journal, 75, pp. 793-517.

Graham, J.W. and C.A. Green (1984), "Estimating the parameters of a household production function with joint products". The Review of Economics and Statistics, 66, pp. 277-282.

Gronau, R. (1977), "Leisure, home production, and work; the theory of the allocation of time revisited". Journal of Political Economy, 85, pp. 1099-1123.

Gronau, R. (1980), "Home production - a forgotten industry". The Review of Economics and Statistics, 62, pp. 408-416.

Kooreman, P. and A. Kapteyn (1987), "A disaggregated analysis of the allocation of time within the household". Journal of Political Economy, 95, pp. 223-249.

Rosenzweig, M.R. and T.P. Schultz (1983), "Estimating a household production function: heterogeneity, the demand for health inputs, and their effects on birth weights". Journal of Political Economy, 91, pp. 723-746.

IN 1986 REEDS VERSCHENEN

202 J.H.F. Schilderinck
Interregional Structure of the European Community. Part III

203 Antoon van den Elzen and Dolf Talman
A new strategy-adjustment process for computing a Nash equilibrium in a noncooperative more-person game

204 Jan Vingerhoets
Fabrication of copper and copper semis in developing countries. A review of evidence and opportunities

205 R. Heuts, J. van Lieshout, K. Baken
An inventory model: what is the influence of the shape of the lead time demand distribution?

206 A. van Soest, P. Kooreman
A Microeconometric Analysis of Vacation Behavior

207 F. Boekema, A. Nagelkerke
Labour Relations, Networks, Job-creation and Regional Development. A view to the consequences of technological change

208 R. Alessie, A. Kapteyn
Habit Formation and Interdependent Preferences in the Almost Ideal Demand System

209 T. Wansbeek, A. Kapteyn
Estimation of the error components model with incomplete panels

210 A.L. Hempenius
The relation between dividends and profits

211 J. Kriens, J.Th. van Lieshout
A generalisation and some properties of Markowitz' portfolio selection method

212 Jack P.C. Kleijnen and Charles R. Standridge
Experimental design and regression analysis in simulation: an FMS case study

213 T.M. Doup, A.H. van den Elzen and A.J.J. Talman
Simplicial algorithms for solving the non-linear complementarity problem on the simplotope

214 A.J.W. van de Gevel
The theory of wage differentials: a correction

215 J.P.C. Kleijnen, W. van Groenendaal
Regression analysis of factorial designs with sequential replication

216 T.E. Nijman and F.C. Palm
Consistent estimation of rational expectations models

232 A.B.T.M. van Schaik, R.J. Mulder
On Superimposed Recurrent Cycles

233 M.H.C. Paardekooper
Sameh's parallel eigenvalue algorithm revisited

234 Pieter H.M. Ruys and Ton J.A. Storcken
Preferences revealed by the choice of friends

235 C.J.J. Huys en E.N. Kertzman
Effectieve belastingtarieven en kapitaalkosten

236 A.M.H. Gerards
An extension of König's theorem to graphs with no odd-K_4

237 A.M.H. Gerards and A. Schrijver
Signed Graphs - Regular Matroids - Grafts

238 Rob J.M. Alessie and Arie Kapteyn
Consumption, Savings and Demography

239 A.J. van Reeken
Begrippen rondom "kwaliteit"

240 Th.E. Nijman and F.C. Palmer
Efficiency gains due to using missing data. Procedures in regression models

241 Dr. S.C.W. Eijffinger
The determinants of the currencies within the European Monetary System

IN 1987 REEDS VERSCHENEN

242 Gerard van den Berg
Nonstationarity in job search theory

243 Annie Cuyt, Brigitte Verdonk
Block-tridiagonal linear systems and branched continued fractions

244 J.C. de Vos, W. Vervaat
Local Times of Bernoulli Walk

245 Arie Kapteyn, Peter Kooreman, Rob Willemse
Some methodological issues in the implementation
of subjective poverty definitions

246 J.P.C. Kleijnen, J. Kriens, M.C.H.M. Lafleur, J.H.F. Pardoel
Sampling for Quality Inspection and Correction: AOQL Performance Criteria

247 D.B.J. Schouten
Algemene theorie van de internationale conjuncturele en strukturele afhankelijkheden

248 F.C. Bussemaker, W.H. Haemers, J.J. Seidel, E. Spence
On (v,k,λ) graphs and designs with trivial automorphism group

249 Peter M. Kort
The Influence of a Stochastic Environment on the Firm's Optimal Dynamic Investment Policy

250 R.H.J.M. Gradus
Preliminary version
The reaction of the firm on governmental policy: a game-theoretical approach

251 J.G. de Gooijer, R.M.J. Heuts
Higher order moments of bilinear time series processes with symmetrically distributed errors

252 P.H. Stevers, P.A.M. Versteijne
Evaluatie van marketing-activiteiten

253 H.P.A. Mulders, A.J. van Reeken
DATAAL - een hulpmiddel voor onderhoud van gegevensverzamelingen